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PROBABILITY DISTRIBUTION FUNCTIONS OF RANDOM VARIABLES

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Abstract: to characterize the properties of random variables in probability theory, the concept of the probability distribution law of a random variable is used. The probability distribution law provides complete information about the properties of a random variable and allows one to obtain the probabilistic value of the measured quantity and the characteristics of the random error. The main characteristics of the probability distribution laws of random variables are the integral and differential distribution functions and the numerical characteristics of the position, dispersion, asymmetry and excess of probability distributions.

Key words: measurement result, probability theory, random variables, random errors, distribution laws, cumulative distribution function, measurement histogram, specified interval.

Introduction. The results of measurements of quantities and the errors in their determination are random variables. This circumstance predetermines the use of methods of probability theory and mathematical statistics to find quantitative estimates of the measurement result and its error.

To characterize the properties of random variables in probability theory, the concept of the probability distribution law of a random variable is used. The probability distribution law provides complete information about the properties of a random variable and allows one to obtain the probabilistic value of the measured quantity and the characteristics of the random error.

Main Part. The main characteristics of the probability distribution laws of random variables are the cumulative and differential distribution functions and the numerical characteristics of the position, dispersion, asymmetry, and kurtosis of probability distributions.

The cumulative distribution function F(x) of a random variable x represents the dependence of the probability that the result of observation xi in the i-th experiment will be less than some current value x on the variable x itself:

$$F(x) = P\{x_i \leq x\} = P\{-\infty < x_i \leq x\}, \quad (1)$$

where: P is the probability of the event described in curly brackets.

The differential distribution function, otherwise known as the probability density function p(x) of a random variable, is equal to the derivative of the cumulative distribution function:

$$p(x) = d\,F(x)/dx. \quad (2)$$

Thus, the integral and differential distribution functions are also related to each other by the following expression:

$$F(x) = \int_{-\infty}^{x} p(x)\cdot dx\,. \quad (3)$$

The formation of a differential distribution function can be illustrated by the example of measurements with multiple observations. Let's say n consecutive measurements of the same quantity X are made, and a group of results from these measurements x1, x2, x3, … xn is obtained. All these results are random numbers, since each contains some random error.

Initially, the observation results are arranged in ascending order from xmin to xmax, and the range of the resulting series is found.

$$L = x_{max} - x_{min}$$

By dividing the series range into k equal intervals Δl = L / k, the number of observations nk falling within each interval is calculated. The obtained results are represented graphically, with the values of the quantity plotted on the abscissa and the interval boundaries indicated, and the relative frequency of observations falling within each interval – nk/n – plotted on the ordinate.

By plotting a rectangle on the diagram, the base of which is the interval width and the height of which is the frequency nk/n, we obtain a so-called histogram, a figure that provides a visual representation of the distribution density of observations in a given experiment.

Figure 1 shows a histogram obtained in one of the experiments and constructed based on the results of 50 observations grouped in Table 1. In the given example, 0.1; 0.2; 0.36; 0.22, and 0.12 of the total number of observations fall into the first and subsequent intervals, respectively. Moreover, from the very principle of determining the relative frequency of observation results falling within each interval, it clearly follows that the sum of all these numbers is equal to one.

If the distribution of the values of a random variable x is statistically stable, then we can expect that with repeated series of observations of the same value and under the same conditions, the relative frequencies of occurrences within each interval will be close to the original values. This means that by constructing a histogram once, we can, with a certain degree of certainty, predict the distribution of observation results across intervals in subsequent series of observations.

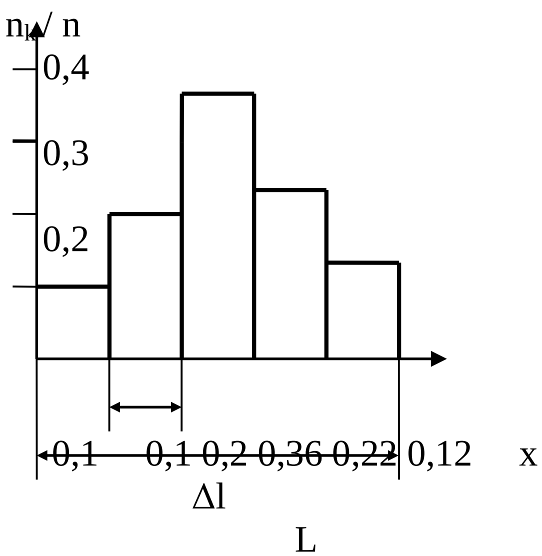

Fig. 1. Histogram

Table 3.1. Initial data for constructing a histogram		
Interval number	n_k	n_k / n
1	5	0,1
2	10	0,2
3	18	0,36
4	11	0,22
5	6	0,12

Taking the total area limited by the histogram contour and the abscissa axis as a unit (S0 = 1), the relative frequency of occurrence (nk/n) of observation results in a given interval can be determined as the ratio of the area of the corresponding rectangle of width Δl to the total area.

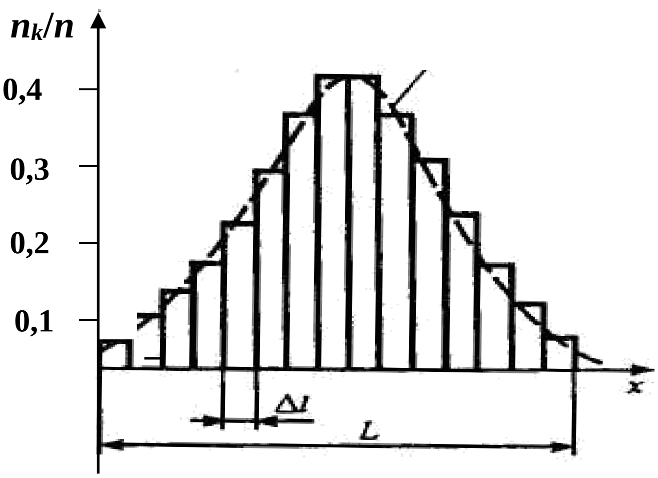

Fig. 2. Probability distribution density curve

With an infinite increase in the number of observations n → ∞ and an infinite decrease in the interval width Δl → 0, the step curve enveloping the histogram will transform into a smooth curve p(x) (Fig. 2), called the probability density curve of the random variable, and the equation describing it is called the differential distribution law. The probability density curve is always non-negative and the area bounded by the curve and the abscissa axis is equal to one..

The value of the integral distribution function for x → - ∞ is equal to zero, and for x → ∞ it is equal to one, i.e.

$$F(-\infty) = 0, \quad F(+\infty) = 1.$$

Hence

$$P\{-\infty < x \le +\infty\} = \int_{-\infty}^{+\infty} p(x)dx = 1.$$

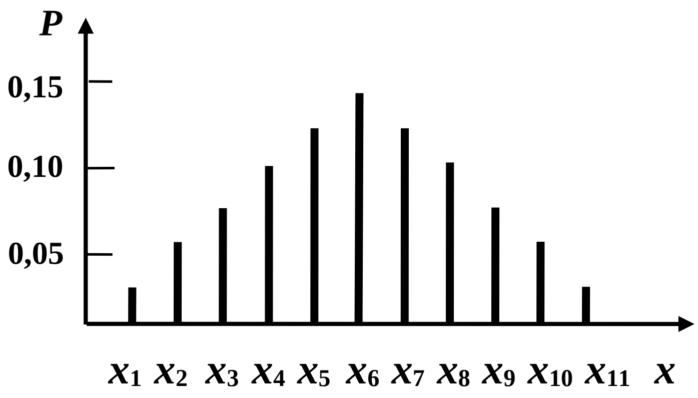

Fig. 3 Distribution of a discrete random variable

The probability of an observation result or a random error falling within a given interval [x1; x2] is equal to the difference in the values of the integral distribution function at the boundaries of this interval

$$P\{x_1 < x \le x_2\} = F(x_2) - F(x_1) = \int_{x_1}^{x_2} p(x)\cdot dx. \quad (4)$$

Graphically, this probability is expressed as the ratio of the area lying under the curve p(x) in the interval from x1 to x2 to the total area bounded by the distribution curve.

In addition to continuous random variables, discrete random variables are also encountered in metrological practice. An example of the distribution of a discrete random variable is shown in Figure 3.

Conclusion. As a result of our theoretical analyses, we found that the main characteristics of the probability distribution laws of random variables are the integral and differential distribution functions and the numerical characteristics of the position, dispersion, skewness, and kurtosis of probability distributions.

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