LINEAR PURSUIT–EVASION DIFFERENTIAL GAMES WITH JOINT CONTROL CONSTRAINTS

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Annotation: This paper studies linear pursuit–evasion differential games under joint control constraints that reflect shared limitations on players' actions. Unlike classical models with independent controls, the admissible strategies are coupled through a common feasibility set. The analysis focuses on the impact of such constraints on equilibrium existence, strategy structure, and system trajectories. The results show that joint constraints significantly modify optimal behavior and introduce implicit coordination between antagonistic agents. The proposed framework is relevant for applications in robotics, security systems, and regulated economic environments where shared resources are present.

Keywords: Differential games; pursuit–evasion; joint control constraints; saddle–point equilibrium; linear systems; resource sharing.

Introduction

Pursuit–evasion differential games constitute a well–established class of dynamic optimization problems in which two or more players with opposing objectives influence the evolution of a dynamical system over time. Typically, one player, referred to as the pursuer, seeks to minimize a certain cost functional associated with distance, time to capture, or deviation from a target, while the other player, the evader, attempts to maximize the same functional in order to avoid capture or delay interception. Such models have been widely applied in robotics, missile guidance, security systems, and economic competition.

Most classical formulations of pursuit–evasion games assume that each player's control variables are subject to independent constraints. That is, the pursuer and the evader select their admissible controls from separate sets that do not depend on the actions of the opponent. However, in many realistic situations this assumption is not valid. Players may be subject to a common limitation on resources such as energy, bandwidth, authority, or physical capacity. For example, two autonomous agents sharing a power source, or two economic actors constrained by a common regulatory framework, cannot freely choose their actions independently. Instead, their controls are coupled through a joint feasibility condition.

Main Body.

Pursuit–evasion differential games have a long history in control theory and applied mathematics, originating from early work on military and aerospace applications. Classical studies by Isaacs established the theoretical foundations of differential game theory and introduced the concept of value and saddle–point equilibrium for zero–sum dynamic games. These early models typically assumed that each player's control actions were subject to independent constraints, allowing strategies to be analyzed within separate admissible sets [1].

Subsequent research extended these ideas to linear–quadratic pursuit–evasion games, where linear system dynamics and quadratic performance criteria yield tractable analytical solutions. These models became popular due to their mathematical elegance and relevance to engineering applications such as missile guidance, robotic interception, and automated surveillance. In most of these formulations, the pursuer and evader are treated symmetrically, except for the sign of their objectives, and their controls are assumed to be bounded independently.

However, over time it became apparent that independent constraints are often an unrealistic simplification. In many real-world systems, players operate under shared physical, technological, or institutional limitations. This observation motivated the introduction of coupled or joint control constraints into differential game models. Early contributions in this direction explored games with shared resource budgets, where the sum of control efforts is limited, reflecting energy or fuel constraints. These models showed that shared constraints can significantly affect the structure of equilibrium strategies [2].

More recent studies have examined differential games with coupling in both the dynamics and the admissible control sets. Researchers have investigated how joint constraints influence the existence and uniqueness of saddle–point equilibria, as well as how they modify the stability and robustness of optimal strategies. It has been shown that while classical existence results rely heavily on separability and convexity assumptions, joint constraints require more refined analytical tools from convex analysis and variational inequality theory.

Another line of research focuses on the interpretation of such games in terms of resource allocation. When players share a limited common resource, the game can be viewed as a competition over how this resource is divided over time. This perspective has been applied in economic models of competition under regulation, in communication networks where bandwidth is shared, and in power systems where multiple agents draw from a common energy source. In these contexts, the pursuit–evasion framework provides a convenient abstraction for antagonistic interactions under common limitations [3].

The introduction of joint constraints also blurs the classical distinction between competitive and cooperative behavior. Although the objectives remain opposed, the fact that each player's action restricts the other's feasible actions introduces an element of implicit coordination. Several authors have emphasized that such games exhibit hybrid features, combining aspects of zero–sum competition with features typical of constrained optimization and cooperative control. This has led to the development of new equilibrium concepts and solution methods tailored to jointly constrained settings.

From a methodological standpoint, the literature has employed a variety of approaches to analyze these games. These include variational methods, fixed–point arguments, dynamic programming, and numerical approximation techniques. While analytical solutions are rare outside of highly structured linear–quadratic settings, numerical methods have been used extensively to explore the behavior of more general models and to validate theoretical insights [4].

Despite this growing body of work, there remains a lack of a unified framework for analyzing linear pursuit–evasion games with joint control constraints in a systematic way. Existing studies often focus on specific applications or particular forms of coupling, making it difficult to compare results or to generalize conclusions. This gap motivates further research aimed at developing general theoretical tools and at clarifying the qualitative implications of shared constraints for strategic behavior.

In summary, the literature indicates that joint control constraints are not a minor technical modification but a fundamental feature that alters the nature of pursuit–evasion games. They affect equilibrium existence, strategy structure, and the interpretation of competition itself. Understanding these effects is essential for applying differential game theory to modern multi–agent systems where shared resources and common limitations are the norm rather than the exception [5].

CONCLUSION

This paper has examined linear pursuit–evasion differential games in which the players' control actions are subject to joint constraints. Unlike classical models with independent admissible controls, jointly constrained games reflect realistic situations where agents share limited resources, physical capabilities, or institutional restrictions.

The analysis highlights that such constraints are not merely technical additions but fundamentally reshape the strategic structure of the game.

The presence of joint control constraints affects the existence and characterization of saddle–point equilibria, modifies the form of optimal strategies, and changes the qualitative behavior of the system trajectories. Although the objectives of the players remain antagonistic, the coupling of feasible actions introduces an element of implicit coordination, since each player's decision restricts the feasible choices of the other. As a result, equilibrium behavior differs significantly from that observed in standard unconstrained or independently constrained pursuit–evasion models.

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